

The matrix elements of the hamiltonian are

$$\begin{aligned} \langle \mathbf{k} + \mathbf{G} | H - \epsilon | \mathbf{k} + \mathbf{G}' \rangle &= \left(-\frac{\hbar^2}{2m}(\mathbf{k} + \mathbf{G})^2 - \epsilon \right) \delta_{\mathbf{G}\mathbf{G}'} + \sum_{\mu} S_{\mu}(\mathbf{G} - \mathbf{G}') \left(V_{\mu}(\mathbf{G} - \mathbf{G}') + \sum_i V_{\mu,i}(\mathbf{k} + \mathbf{G}, \mathbf{k} + \mathbf{G}') \right) \\ &+ V_{Hartree}(\mathbf{G} - \mathbf{G}') + V_{xc}(\mathbf{G} - \mathbf{G}'). \end{aligned} \quad (16)$$

Divergent Terms in the potential

The Hartree term, $V_{Hartree}(\mathbf{G})$, and local potential term, $\sum_{\mu} S_{\mu}(\mathbf{G})V_{\mu}(\mathbf{G})$, are separately divergent at $\mathbf{G} = 0$ and must be treated in a special way. Let us consider their sum $\tilde{V}(\mathbf{r}) = V_{loc}(\mathbf{r}) + V_{Hartree}(\mathbf{r})$. Its $\mathbf{G} = 0$ term is not divergent:

$$\tilde{V}(\mathbf{G} = 0) = \frac{1}{\Omega} \int d\mathbf{r} \left(\sum_{\mu} V_{\mu}(\mathbf{r} - \mathbf{d}_{\mu}) + \frac{1}{N} e^2 \int \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' \right) = \frac{1}{\Omega} \sum_{\mu} \int d\mathbf{r} \left(V_{\mu}(r) + \frac{Z_{\mu} e^2}{r} \right) = \frac{1}{\Omega} \sum_{\mu} \alpha_{\mu} \quad (17)$$

where we used

$$V_{\mu}(r) \sim -\frac{Z_{\mu} e^2}{r} \quad \text{for large } r, \quad \frac{1}{N} \int n(\mathbf{r}) = \sum_{\mu} Z_{\mu}. \quad (18)$$

The α_{μ} are parameters depending only on the pseudopotential.